

Self-organized Trail Systems in Groups of Humans

ROBERT L. GOLDSTONE AND MICHAEL E. ROBERTS

Indiana University, Bloomington, Indiana 47405

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We have developed an experimental platform for studying the trail systems that spontaneously emerge when people are motivated to take advantage of the trails left by others. In this virtual environment, the participants' task is to reach randomly selected destinations while minimizing travel costs. The travel cost of every patch in the environment is inversely related to the number of times the patch was visited by others. The resulting trail systems are a compromise between people going to their destinations and going where many people have previously traveled. We compare the results from our group experiments to the Active Walker model of pedestrian motion from biophysics. The Active Walker model accounted for deviations of trails from the beeline paths, the gradual merging of trails over time, and the influences of scale and configuration of destinations on trail systems, as well as correctly predicting the approximate spatial distribution of people's steps. Two deviations of the model from empirically obtained results were corrected by (1) incorporating a distance metric sensitive to canonical horizontal and vertical axes, and (2) increasing the influence of a trail's travel cost on an agent's route as the agent approaches its destination. © 2006 Wiley Periodicals, Inc. Complexity 11: 43–50, 2006

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What kind of trail systems do people spontaneously form when they are motivated to take advantage of the trails left by their predecessors? In the process of exploiting previously left trails, people further reinforce

Correspondence to: Robert Goldstone, E-mail: rgoldsto@indiana.edu

these trails, potentially leading to lock-in of originally tentative and faint paths. Early trail blazers through a jungle use machetes to make slow progress in building paths—progress that is capitalized on and extended by later trekkers, who may then widen the trail and then later put stones down, then gravel, and then asphalt. A distinguished proponent of this kind of emergent trail system was Dwight Eisenhower, who, as president of Columbia University was

FIGURE 1



Example of a spontaneous path formed by pedestrians at Indiana University, leading to the house of the president of the university. Notice that groundskeepers have felt impelled to reinforce the self-organized pathway by laying down stones at its beginning (Photograph by Robert Goldstone).

asked how the university should arrange the sidewalks to best interconnect the campus buildings. He responded that they should first plant grass seed, let the grass grow, see where the grass became worn by people's footsteps and install the sidewalks in the most worn patches.

Figures 1 and 2 show examples of spontaneously created pedestrian paths at our university. Figure 1 demonstrates the process of path strengthening near the president's house. The spontaneously created footpath has been reinforced by a stone path laid down by groundskeepers near where the footpath meets the president's driveway. The footpath preceded and impelled, the construction of the stone path. If the stone path succeeds in facilitating further travel, then we would expect it to divert even more pedestrian traffic to it, leading to even sturdier paths. Path systems are a concrete example of the general principle that activity often begets more activity. Other examples of this principle includes sales of a popular music CD leading to further sales of the CD because of the media attention garnered by its popularity [1], and a scholarly paper receiving citations because other authors have cited the paper [2].

Figure 2 shows that even when highly developed path systems exist, pedestrians often prefer to travel less developed pathways that lead more directly to their destinations. Spontaneously produced paths tend to be a compromise between pedestrians moving in a beeline path toward their destination and moving along well-established paths. Even though the pedestrian in Figure 2 eschews the major roads, he is still traveling along a worn footpath established by earlier pedestrians. Advantages of traveling along existing paths include 1) reduced physical effort (important for

FIGURE 2



Another spontaneously created path at Indiana University. Travelers may occasionally eschew established road systems (the red brick path and asphalt road above) and forge their own new paths. However, once forged, the paths are often attractive for subsequent travelers, who further reinforce these paths. Following a worn path reduces the physical and cognitive costs for travelers.

travel in dense jungles), 2) increased comfort (e.g., following another person's footsteps while traveling in the snow), and 3) decreased cognitive cost (e.g., rather than consulting a map, a traveler may follow a well-worn path, figuring that it will lead to the desired destination). Human social structures are highly adaptable and efficient because people will sometimes forge their own new paths when existing paths are poorly tuned to their needs. It often also turns out that these newly forged paths will be attractive to subsequent travelers, who will reinforce and extend these paths. The main agenda of this article is to describe an experiment on group path formation in a controlled laboratory setting and to present a computational model of the self-organized nature of the resulting trail systems.

A COMPUTATIONAL MODEL OF TRAIL FORMATION

The biophysicist Dirk Helbing and his colleagues [3–5] have developed a computational model of self-organized trail formation that assumes that agents move in their environment, leaving trails as they do so. Their agent-based "Active Walker" model assumes walkers move to destinations, and as they take steps, they affect their environment, facilitating travel for subsequent walkers. Walkers compromise between taking the shortest way to their destination and using existing, strong trails. The core idea is that each spot on a terrain has a potential describing its ease of travel, and the ease of travel of a spot is increased by walkers' steps on it and decreases as the path erodes. The ease of travel of position \mathbf{r} at time t is expressed by $G(\mathbf{r}, t)$ and its change is governed by

$$\frac{dG(\mathbf{r}, t)}{dt} = \frac{1}{T(\mathbf{r})} [G_0(\mathbf{r}) - G(\mathbf{r}, t)] + I(\mathbf{r}) \left[1 - \frac{G(\mathbf{r}, t)}{G_{\max}(\mathbf{r})} \right] \sum_{\alpha} \delta(\mathbf{r} - \mathbf{r}_{\alpha}(t)), \quad (1)$$

where the δ function is Dirac's delta function, which only is positive for values of 0. $T(\mathbf{r})$ is a parameter reflecting the durability of trails from erosion, and $I(\mathbf{r})$ reflects the influence of steps on changing the ease of travel. The first factor in Equation (1) decreases ease of travel for a location based on how much easier travel is for a location compared with its baseline ease of $G_0(\mathbf{r})$, which is 0 in our simulations. The second factor increases ease of travel as a function of the trodden upon location's current ease compared to its maximally possible ease, $G_{\max}(\mathbf{r})$. The attractiveness of a trail segment at location \mathbf{r} from the perspective of an agent at \mathbf{r}_{α} is formalized as a potential function and is based on its proximity to Agent α and an integrated spatial average of the locations' individual comfort levels:

$$V_{tr}(\mathbf{r}_{\alpha}, t) = \int d^2r e^{-|\mathbf{r}-\mathbf{r}_{\alpha}|/\sigma(\mathbf{r}_{\alpha})} G(\mathbf{r}, t). \quad (2)$$

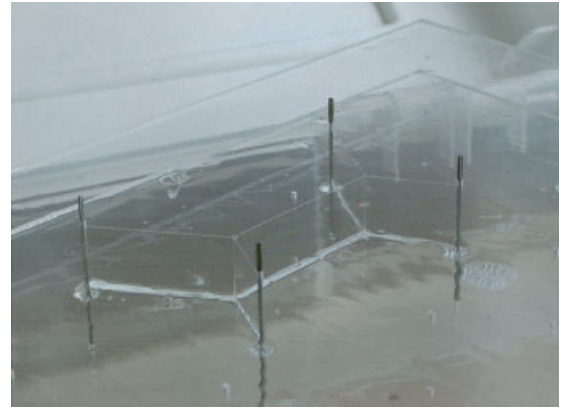
Thus, the trail potential will be influenced by every location's ease of travel, but these influences drop off as an exponential function of their distance to an agent. The $\sigma(\mathbf{r}_{\alpha})$ term reflects Agent α 's visibility—the extent to which it is influenced by distant cells. As visibility increases, the agent will make increasingly large deviations away from beeline paths in order to take advantage of easy-to-travel locations. Finally, the walking direction, e_{α} of agent α is a linear combination of the destination and the trail potential, divided by a factor that normalizes the direction:

$$\mathbf{e}_{\alpha}(\mathbf{r}_{\alpha}, t) = \frac{d_{\alpha} - \mathbf{r}_{\alpha} + \nabla_{\mathbf{r}_{\alpha}} V_{tr}(\mathbf{r}_{\alpha}, t)}{|d_{\alpha} - \mathbf{r}_{\alpha} + \nabla_{\mathbf{r}_{\alpha}} V_{tr}(\mathbf{r}_{\alpha}, t)|}. \quad (3)$$

The d_{α} term is the location of the destination. Before any trails have been formed, walkers will simply take beeline paths to their destinations. However, as trails begin to be formed, walkers will often take detours to take advantage of the increased comfort of the trails, thereby further reinforcing the comfortable trails. The experiment that we will describe, although developed independently of the Active Walker model, almost precisely concurs with the assumptions of this model. Accordingly, one of the goals of the experiment is to compare this model to the trails left by humans interacting in a virtual world.

A useful baseline model for comparing both the human data and the Active Walker model is provided by Minimal Steiner Trees (MSTs). A MST is the set of paths that connects a set of points (e.g., destinations) using the minimal amount

FIGURE 3



When a four-pin arrangement is dipped into a soap bath, a Minimal Steiner Tree (MST) is created from the soap film that connects the pins. In this case, the MST has two Steiner points with three equally spaced paths coming out of each Steiner Point. As shown here, Steiner points can connect to other Steiner points (Photograph by Robert Goldstone).

of total path length. MSTs feature new points, called Steiner Points, in addition to the original destination points. Soap films frequently create patterns that minimize surface tension, resulting in MST configurations [6]. Figure 3 show a device that can be easily constructed to study soap film formation. It features two plastic, parallel plates with a gap between them and with small holes for inserting pins. If the device is dipped into a soap bath and removed, soap film will remain on the device, clinging to, and connecting the pins. Figure 3 shows that Steiner Points may connect to other Steiner Points rather than destinations. In the 1600s, Fermat and Toricelli showed that there are always three lines coming out of a Steiner point, and each pair of adjacent lines establishes a 120° angle. Finding MSTs is a notorious NP-complete problem, with all known, provably optimal algorithms requiring an exponential increase in computation as the number of destination points increases linearly [7].

Given the computational difficulty of finding MSTs, it is noteworthy that soap films often find these solutions without any explicit search or backtracking. The existence proof of soap film MSTs leaves open the possibility that other complex systems that operate according to different principles, including groups of people, might also approximate MSTs. MSTs provide one formal measure of the large-scale patterns and allows us to quantify the efficiency of self-organized group patterns as well as their systematic deviations from optimal patterns. Our primary interest is not in whether individuals can explicitly solve MST problems. Prior work has shown that individuals can find surprisingly good approximations to MST solutions [8] as well as to the

FIGURE 4

Sample screen display during an experiment with four participants. Participants see themselves as green triangles. All of the other participants appear as yellow dots. Cities are shown as blue dots, except for the participant's destination city which appears in green. The cost of every screen location is coded by its brightness using a red palette, with brighter locations signifying lower costs.

related Traveling Salesperson Problem [9]. Instead, our work follows in the tradition of studying whether and when decentralized systems like groups of ants [10] can approximate MSTs.

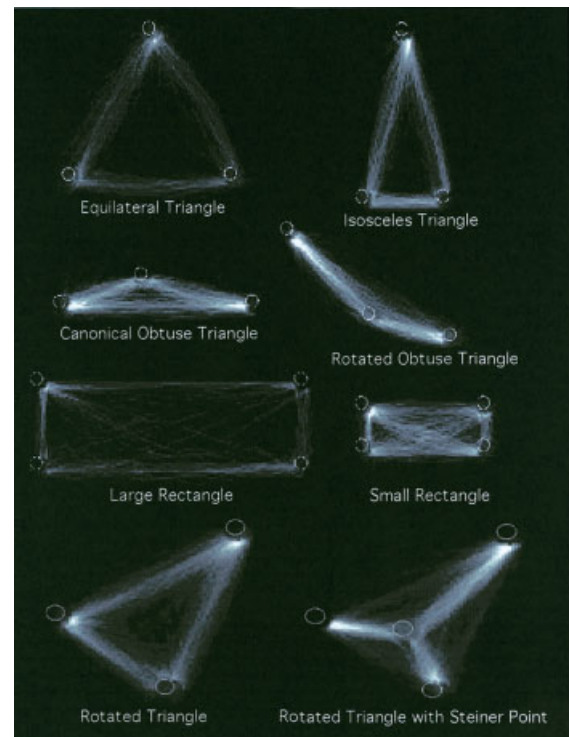
To study this question, we adapted a software platform that we have developed to conduct other multiparticipant experiments over the Internet [11–13]. The software allows us to have moderate group sizes of up to 100 people interacting simultaneously in a virtual environment, viewing the moment-by-moment behaviors of themselves and the other participants. Using this software, we developed a virtual environment in which people move between destinations and are motivated to take advantage of each others' paths.

A PSYCHOLOGICAL EXPERIMENT ON GROUP TRAIL FORMATION

Methods

Three hundred and two undergraduate students from Indiana University served as participants in order to fulfill a course requirement. The students were run in 34 groups with 6–12 participants per group.

The participants were all physically located in the same room for experimental control. The participants were instructed that their task was to travel from city to city while earning the most points possible. Each participant's point total was shown in the upper-left corner of the screen. They were instructed that they would receive a certain number of points for every destination city reached, but that they would lose points for every step taken. They were also instructed that each

FIGURE 5

The cumulative steps taken on each cell of the eight configurations. The brightness of a location is proportional to the number of times that it was stepped on by all participants. The destinations are indicated by white circles.

cell of the 150×100 world would have its own changing cost, and that a cell's cost decreased as a function of the number of times it had been trodden. The instantaneous cost of a cell is shown by its brightness, with more expensive cells appearing darker, as shown in Figure 4.

Each experiment was divided into 5-min periods of travel across the eight configurations in Figure 5. Each configuration consisted of 3 to 4 cities, shown as large blue dots in Figure 4. At any time, each participant was randomly assigned one of the cities as their destination, and this destination was colored green. When participants reached their destination city, a new and different destination city was selected, and the participant was given a number of points equal to 15 times the distance between their starting and destination cities. Participants began each configuration at uniformly distributed random locations.

Participants appeared as yellow dots, except for a participant's own position which appeared on the screen by a green triangle, with the most acute corner of the triangle denoting the participant's orientation. The heading direction was altered by pressing the computer keyboard's right arrow key to move clockwise by 3° , and the left arrow to

move counter-clockwise by 3°. Participants moved forward by pressing the up arrow key, and kept moving until they pressed the down arrow key. When participants reached their destination, they automatically stopped and needed to press the up arrow to once again move forward.

Every step taken by every participant influenced the travel cost of each cell of the world. Although participants' steps decreased travel cost, travel cost increased over time to a maximum value to model the regrowth of obstructions and the erosion of path systems. Travel ease for each cell gradually decreased by $N\%$ every second, where N is the number of participants. Tying path erosion to the number of participants roughly equated the overall travel ease for groups of different size. When a participant stepped on a cell, the cost decrease was diffused to neighboring squares according to a Gaussian distribution. Specifically, the ease of travel of every cell was increased by $C_1 \exp(-C_2 D_{P,L}^2)$, where $D_{P,L}$ is the Euclidean distance between Participant P and a cell's location L , C_1 is a parameter controlling the influence of a participant on cells' travel cost, and C_2 is a parameter controlling the extent of diffusion to neighbors. For our experiments, C_1 was 0.007 and C_2 was 0.18. Figure 4 shows how the moment-by-moment changes to cells' travel costs were reflected by the brightness of the cells.

Participants also received continual updates of other participants' positions at least once every 2 seconds, their own score, and the instantaneous cost of stepping on their current cell. The instantaneous cost was shown graphically by the length of a blue bar at the top of the screen (see the top of Figure 4).

RESULTS

For each of the eight configurations of cities, the number of times every location was stepped on is indicated by its brightness in Figure 5. The locations of the cities are indicated by circles. Configurations that were designed to be compared with one another are placed side by side in Figure 5. One immediately apparent result is that participants do not generally create MSTs. However, there are deviations from beeline paths between cities, and frequently these deviations are in the direction toward MST paths. In what follows, such deviations will be referred to as "pro-Steiner deviations." There was apparently more pro-Steiner deviation from beeline pathways for the isosceles than equilateral triangle. This is indicated by the large bright area directly below the top city for the isosceles but not equilateral triangle. This is intriguing because it can be proven that the largest total savings of path distance for the MST over the spanning tree or the set of beeline paths is found for the equilateral triangle, not the isosceles triangle. Having a particularly advantageous optimal path is no guarantee that a group will find it. For the equilateral triangle, there is little incentive for a pioneer to move straight down from the top city; the terrain is dark and costly there (see Figure 5). For the

isosceles triangle this area becomes brighter because of the overlapping diffusion from the two beeline paths from the top city, and once people go straight down, it is attractive for others, and so the vertical path is extended still further.

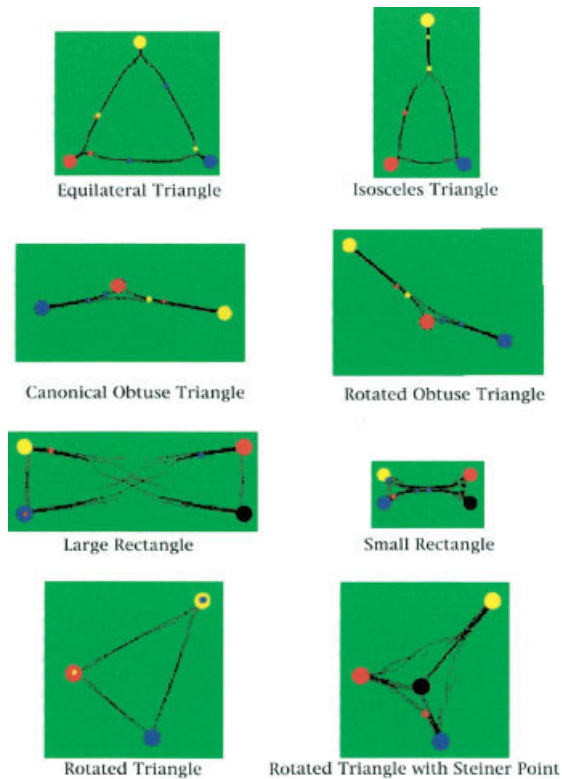
For the second pair of configurations, the same obtuse 150° triangle is presented in two orientations. In the canonical obtuse configuration, the longest path is a horizontal line. For the rotated obtuse, the longest path has an undistinguished orientation of 122°. For both configurations of points, the MST is identical to the minimal spanning tree and is simply the combination of the two short paths. Figure 5 shows that there is more travel close to the beeline path connecting the furthest cities for the canonical than rotated obtuse. Prior research has supported a privileged status for horizontal and vertical axes [14–16]. It is plausible that participants gravitate toward these canonical axes because of their status as privileged frames of reference.

For the third pair of configurations, two rectangular configurations of cities have identical aspect ratios, differing only in their scaling factor. Figure 5 shows more deviation of the outer paths toward the MST for the small than large rectangle. A similar explanation may be at work here as for the isosceles triangle. Pronounced deviations from beeline paths are observed when two paths are close enough that a path created for one purpose is available to be used for another purpose.

The final pair of configurations compares an isosceles triangle to the same isosceles triangle with its Steiner point included as a destination point. When the Steiner point is added to the triangle, all of the other paths deviate more toward the Steiner point. In fact, for this path system, people do frequently use the MST path to reach any pair of cities, which is why the outer beeline paths are so dim. Early steps along the paths involving the Steiner point city are later exploited by participants traveling among the outer cities. This, in turn, makes the outer paths still dimmer, prompting even greater use of the MST path.

The plots in Figure 5 are imprecise because they do not distinguish which pair of cities a person was traveling between. For our statistical analysis, we considered each participant's journey from one city to another. For each step on this path, we measured the distance of the step to the beeline path and coded it as positive if the deviation was in the direction of the MST path and negative if it was in the opposite direction. The scalar measure of a participant's beeline path deviation is an integration of the step-by-step deviations.

Using this measure of integrated deviation, we quantitatively confirmed the previous conclusions. This technique has the added advantage of allowing us to look at asymmetries, where people take different paths from A to B than from B to A. In fact, the isosceles triangle had a significant asymmetry for paths, with the paths to the top point being significantly more pro-Steiner than the paths from the top point. In addition, the canonical obtuse triangle has signif-

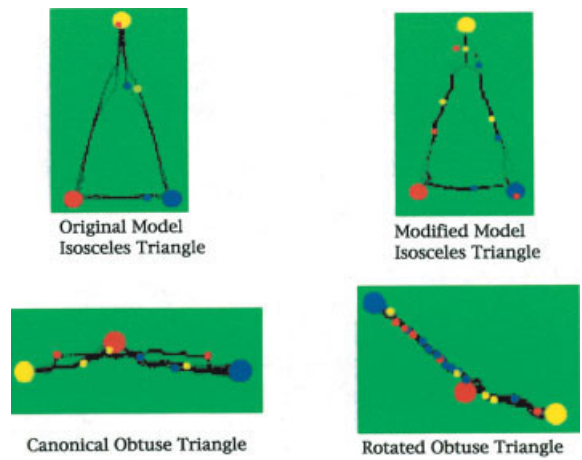
FIGURE 6

Sample screen plots of the Active Walker model [3] for the destination configurations used in our experiment. Destinations are shown as large circles, and individuals are shown as small dots whose color matches the color of their destination city. The darkness of a cell is positively related to its level of comfort for walking.

icant path asymmetries, with paths to the upper, central point deviating significantly more from the beeline path than paths going from the upper point. Both of these asymmetries can be explained by participants being more willing to deviate from beeline paths at the end of a journey than at the beginning. For example, both of the paths to the top point of the isosceles triangle in Figure 5 are more pro-Steiner than the paths going away from the top point. This is predicted if people are more likely to be attracted by the low-cost region underneath the top point on their way to the point. On the way from the point, people may not want to take an immediate detour to take advantage of the low-cost region. This heuristic is an extension of the “road climbing” heuristic [17,18] by which there is a preference for long and straight routes in the local area containing the origin, as opposed to destination, of a trail.

MODELS OF THE EMPIRICALLY OBTAINED TRAIL SYSTEMS

Varying only the single parameter $\sigma(\mathbf{r}_a)$ that controls the visibility (hence influence) of paths within the Active Walker

FIGURE 7

Sample screen plots from our modified Active Walker model, compared with the original model with equivalent parameters. Destinations are shown as large circles, and individuals are shown as small dots whose color matches the color of their destination city. The original and modified models both predict path asymmetries for the isosceles configuration, but in opposite directions. As shown in the lower panels, the modified model no longer predicts rotational invariance.

Model, the model is able to broadly predict several aspects of our results. Sample screen plots from the model are shown in Figure 6. Consistent with our experimental results, the model does not generally predict MST trails but rather trails that deviate away from beeline paths toward MST paths. The model correctly predicts more pro-Steiner deviation for the isosceles than equilateral triangle. Moreover, for the isosceles triangle, the model correctly predicts that the greatest deviation from beeline paths is found near the top triangle point (compare to Figure 5). It also correctly predicts greater pro-Steiner deviation with passing time. Over time, the originally separated beeline routes become progressively zipped together, starting at the top point.

The model correctly predicts greater pro-Steiner deviation from beeline paths for small than large rectangles, for the same reason it predicts greater deviations for the isosceles than equilateral triangle. Paths are closer to one another for the small than large rectangle, and consequently there is more opportunity for re-use of close paths.

The model correctly predicts greater pro-Steiner deviation from beeline paths when the Steiner point is added to an isosceles triangle, because the paths to the Steiner point become attractive candidates for re-use by walkers going to destinations on the outer triangle.

Although the model predicts the general appearance of the obtuse triangle trails, it is rotationally invariant. Consequently, the original model does not predict differences in

trails for the canonical and rotated obtuse triangles. This is shown in Figure 6 by the equivalent trails found by the model for the canonical and rotated obtuse triangle configurations. However, a small change to the model suffices to accommodate our empirical results. A generalized Minkowski distance function is used to calculate the distance between a walker and a location. So, in the calculation of the distance between an agent and a trail segment in Equation (2) above, we replaced the Euclidean distance calculation with a generalized distance of

$$D(\mathbf{r}, \mathbf{r}_\alpha) = [(\mathbf{r}_x - \mathbf{r}_{\alpha x})^s + (\mathbf{r}_y - \mathbf{r}_{\alpha y})^s]^{1/s}, \quad (4)$$

where x and y are indexes for the horizontal and vertical coordinates of the agents and trails. If $s = 2$, then a standard Euclidean distance function is implemented and the resulting path systems are always rotationally invariant. If $1 \leq s < 2$, then the walker will be more strongly influenced by horizontal and vertical trails than diagonal trails at an equal Euclidean distance because the horizontal and vertical trails will be closer by the distance function and hence more visible by Equation (2). Figure 7 shows the resulting path systems for the canonical and rotated obtuse triangles when $s = 1$ (a “city-block” rather than Euclidean metric). Consistent with our human groups, the beeline path connecting the farthest removed destinations is more attractive when this path lies along the horizontal axis rather than an arbitrary diagonal line. For the Rotated Obtuse Triangle almost all of the agents’ steps are directly on the MST path, whereas for the Canonical Obtuse Triangle some of the steps are pulled toward the horizontal path. Thus, by using a $s = 1$ rather than $s = 2$ metric, greater deviations from MST paths and greater conformity to beeline paths is found when the beeline path is horizontal. However, exactly the opposite effect would be found if the MST path and not the longest path connecting destinations was laid along a canonical horizontal or vertical axis.

Finally, the standard Active Walker does predict route asymmetries, but they are in the opposite direction compared with our results. For example, in Figure 7, for the plot labeled “Original Model Isosceles Triangle” there are slightly different paths connecting the yellow and blue cities, as well as the yellow and red cities. The walker going to the yellow city from the blue city is taking the outer path, whereas the walker going from the yellow city to the blue city is following the inner path. Conversely, our results showed greater pro-Steiner deviation going from the blue (or red) to yellow city than vice versa. One relatively simple way of modifying the Active Walker model that is consistent with the “road climbing” heuristic [17,18] is to dynamically alter the path visibility parameter over the course of a journey from one city to another. We do this by replacing $\sigma(\mathbf{r}_\alpha)$ in Equation (2) with $\sigma(\mathbf{r}_\alpha, q)$, the visibility of Agent α at location q , defined as

$$\sigma(\mathbf{r}_\alpha, q) = \left(\frac{\sigma(\mathbf{r}_\alpha)}{1 + e^{2D(d_\alpha, \mathbf{r}_\alpha)}} \right), \quad (5)$$

where D is a Euclidean function of the distance between its arguments—the agent’s current location and its destination. By Equation 5, the effective visibility that an agent has increases as it approaches its destination. This entails that an agent will be more influenced by cells’ ease-of-travel as its journey continues. Figure 7 shows the resulting trail system with the modified model’s change to visibility, demonstrating the same path asymmetry shown by our human participants. This change also allows the model to explain the path asymmetry found with the Canonical Obtuse Triangle. Empirically, we found more pull toward the horizontal pathway for agents going from the top, red destination to the yellow and blue destinations, compared to agents going from yellow and blue destinations to the red destination. This is exactly the behavior shown by the modified model in Figure 7, because deviations from the short beeline paths toward the horizontal path will be more pronounced near the end of a journey than at the beginning.

CONCLUSIONS

To a first approximation, our group behaviors are well modeled by the Active Walker model from biophysics. The model can account for differences in scales and topologies of destinations, the influence of time on emerging paths, and the approximate distribution of actual steps. There were two systematic discrepancies between the model and our empirical results. First, the original model, but not our participants, was unaffected by rotating the configuration of destinations. This difference was removed by incorporating a distance metric in the model that preserves the notion of canonical horizontal and vertical axes. The second discrepancy involved asymmetries and was corrected by having the model’s walkers, like our participants, start their journeys taking the most direct, beeline path toward their destinations and then gradually become more attracted by low-cost detours. These model changes are valuable for creating a psychologically plausible model, but the original Active Walker model’s essential assumption generally fits our experimental results well: people’s movements are a compromise between going where they want to go and going where others have gone.

The currently reported research has been limited to empirically establishing and modeling the dynamics of path formation in human collectives. However, the potential promise of this work is that similar processes are involved in concrete and abstract collective path formation. One generalization is that both involve stigmergy, a form of indirect communication between agents that is achieved by agents modifying their environment and also responding to these modifications [19]. This effect has been well documented in

ant swarms, in which ants lay down pheromones as they walk that attract subsequent ants [20]. Stigmergy has recently been proposed as an important mechanism for achieving multirobot cooperation [21] and robustly interacting software systems [22].

Ideational stigmergic effects occur when a community converges on similar choices because of a beneficial and sensible propensity to take advantage of others' efforts. As with our concrete and spatially instantiated trails, later choosers frequently take advantage of the efforts of earlier choosers. There are often intrinsic advantages to choosing options that are popular. When VHS became more popular than Beta as a format for video recording, then wise people chose VHS because the popularity of VHS led to more movie titles being released on VHS. A similar popularity advantage explains why the QWERTY keyboard continues to be chosen by most people despite its demonstrated inferiority to other keyboard arrangements. Moore's law is a particularly striking case of ideational stigmergy, where for the last 40 years there has been a constant rate exponential increase in the number of transistors per integrated circuit, largely

because technological advances pursue paths of previous innovations and extend them. Early entrants to business sectors [23] and resource pools [11] shape the territory for future participants. Early attitudes and behaviors within a culture create norms that tend to be continued for subsequent generations even after their extrinsic fitness value is no longer favorable [24]. In fact, much, if not all, of what we know of as culture is created by following and extending the innovations of predecessors [25]. The growth of our collective spatial path systems may reveal principles about how future progress is more generally achieved by exploiting and extending prior innovations.

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